

SDLCQ and String/Field Theory Correspondences*

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String/Field theory correspondences have been discussed heavily in recent years. Here, we describe a testing scenario involving a non-perturbative field theory calculation using the framework of supersymmetric discrete light-cone quantization (SDLCQ). We consider a Maldacena-type conjecture applied to the near horizon geometry of a D1-brane in the supergravity approximation. Numerical results of a test of this conjecture are presented with orders of magnitude more states than we previously considered. These results support the Maldacena conjecture and are within 10-15% of the predicted results. We present a method for using a “flavor” symmetry to greatly reduce the size of the Fock basis and discuss a numerical method that we use which is particularly well suited for this type of matrix element calculation. Our results are still not sufficient to demonstrate convergence, and, therefore, cannot be considered to be a numerical proof of the conjecture. We update our continuous efforts to improve on these results and present some results on the way to higher dimensional scenarios.

1. Introduction

The conjecture that certain field theories admit concrete realizations as string theories on particular backgrounds has caused a lot of excitement in the last years. Originally, the so-called Maldacena conjecture [1] assured that the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory in 3+1 dimensions is equivalent to Type IIB string theory on an $AdS_5 \times S^5$ background. Meanwhile, other string/field theory correspondences have been conjectured. Attempts to rigorously test these conjectures have met with limited success, because our understanding of both sides of the correspondences is usually insufficient. The main obstacle is that at the point of correspondence, we require two conditions to hold which are mutually exclusive. Namely, we want a situation where the curvature of the considered spacetime is small in order to be able to use the supergravity approximation to string theory. One also desires the corresponding field theory to be in a small coupling regime. So far it has been impossible to find such a scenario. We present a way out of this dilemma by relaxing the second requirement and performing a *non-perturbative* calculation on the field theory side. To create a manageable situation, we chose a special string/field theory correspondence in order to apply the non-

perturbative method, namely SDLCQ, at its optimal working point.

SDLCQ, or Supersymmetric Discretized Light-Cone Quantization, is a non-perturbative method for solving bound-state problems that has been shown to have excellent convergence properties, in particular in low dimensions. Therefore, we are looking for a (preferably) two-dimensional field theory, which is conjectured to be equivalent to a string theory. It turns out that the Yang-Mills theory with 16 supercharges in two dimensions [2] has its corresponding string theory partner in a system of D1-branes in Type IIB string theory decoupling from gravity [5]. Since both systems have separately been studied in the literature already, this system is an optimal candidate to study the string/field theory correspondence.

The next step is to find an observable that can be computed relatively easy on both sides of the correspondence. It turns out that the correlation function of a gauge invariant operator is a well-behaved object in this sense. We chose the stress-energy tensor $T^{\mu\nu}$ as this operator and will construct this observable in the supergravity approximation to string theory and perform a non-perturbative SDLCQ calculation of this correlator on the field theory side.

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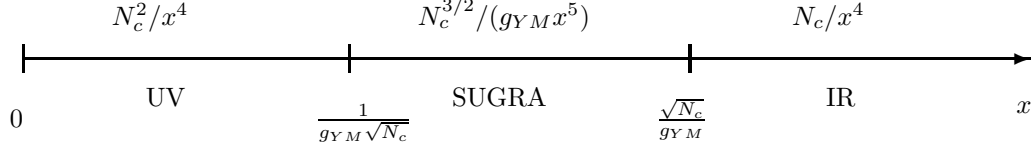


Figure 1. Phase diagram of two-dimensional $\mathcal{N} = (8, 8)$ SYM: the theory flows from a CFT in the UV to a conformal σ -model in the IR. The SUGRA approximation is valid in the intermediate range of distances, $1/g_{YM}\sqrt{N_c} < x < \sqrt{N_c}/g_{YM}$.

2. Correlation functions from supergravity

It is instructive to take a closer look on the expected properties of $\mathcal{N} = (8, 8)$ SYM, before we proceed to technical details on the string theory side. In the extreme ultra-violet (UV) this theory is conformally free and has a central charge $c_{UV} = N_c^2$. Perturbation theory in turn will be valid for small effective couplings $g = g_{YM}\sqrt{N_c}x$, where x is a space coordinate. For large distances, in the far infra-red (IR), the theory becomes a conformal σ -model with target space $(R^8)^{N_c}/S_{N_c}$. The central charge is $c_{IR} = N_c$. It is a bit more involved to show that here perturbation theory breaks down when $x \sim \sqrt{N_c}/g_{YM}$, see *e.g.* Ref. [5].

The intermediate region, $1/g_{YM}\sqrt{N_c} < x < \sqrt{N_c}/g_{YM}$, where no perturbative field theoretical description is possible, is fortunately exactly the region which is accessible to string theory; or rather, to the supergravity (SUGRA) approximation to Type IIB string theory on a special background. It is that of the near horizon geometry of a D1-brane in the string frame, which has the metric

$$\begin{aligned} ds^2 &= \alpha' \hat{g}_{YM} \left(\frac{U^3}{g_s^2} dx_{\parallel}^2 + \frac{dU^2}{U^3} + U d\Omega_{8-p}^2 \right) \\ e^\phi &= \frac{2\pi g_{YM}^2}{U^3} \hat{g}_{YM}. \end{aligned} \quad (1)$$

where we defined $\hat{g}_{YM} \equiv 8\pi^{3/2}g_{YM}\sqrt{N_c}$. In the description of the computation of the two-point function we follow Ref. [3]. The correlator has been derived in Ref. [8], being itself a generalization of Refs. [6,7].

First, we need to know the action of the diagonal fluctuations around this background to the quadratic order. We would like to use the analogue of Ref. [9] for our background, Eq. (1), which is not (yet) available in the literature. However, we can identify some diagonal fluctuating degrees of freedom by following the work on black hole absorption cross-sections [10,11]. One can show that the fluctuations parameterized like

$$\begin{aligned} ds^2 &= (1 + f(x^0, U) + g(x^0, U)) g_{00} (dx^0)^2 \\ &\quad + (1 + 5f(x^0, U) + g(x^0, U)) g_{11} (dx^1)^2 \\ &\quad + (1 + f(x^0, U) + g(x^0, U)) g_{UU} dU^2 \\ &\quad + \left(1 + f(x^0, U) - \frac{5}{7}g(x^0, U) \right) g_{\Omega\Omega} d\Omega_7^2 \\ e^\phi &= (1 + 3f(x^0, U) - g(x^0, U)) e^{\phi_0}, \end{aligned} \quad (2)$$

satisfy the following equations of motion

$$\begin{aligned} f''(U) &= -\frac{7}{U} f'(U) + \frac{g_s^2 k^2}{U^6} f(U) \\ g''(U) &= -\frac{7}{U} g'(U) + \frac{72}{U^2} g(U) + \frac{g_s^2 k^2}{U^6} g(U). \end{aligned} \quad (3)$$

Without loss of generality we have assumed here that these fluctuation vary only along the x^0 direction of the world volume coordinates, and behave like a plane wave. One can interpret a D1-brane as a black hole in nine dimensions. The fields $f(U)$ and $g(U)$ are then the minimal and the fixed scalars in this black hole geometry. In ten dimensions, however, we see that they are really part of the gravitational fluctuation. Consequently, we expect that they are associated with the stress-energy tensor in the operator field correspondence of Refs. [6,7]. In the case of the cor-

respondence between $\mathcal{N}=4$ SYM field theory and string theory on an $AdS_5 \times S^5$ background, the superconformal symmetry allows for the identification of operators and fields in short multiplets [12]. In the present case of a D1-brane, we do not have superconformal invariance and this technique is not applicable. Actually, we expect all fields of the theory consistent with the symmetry of a given operator to mix. The large distance behavior should then be dominated by the contribution with the longest range. The field $f(k^0, U)$ appears to be the one with the longest range since it is the lightest field.

Eq. (3) for $f(U)$ can be solved explicitly

$$f(U) = U^{-3} K_{3/2} \left(\frac{\hat{g}_{YM}}{2U^2} k \right), \quad (4)$$

where $K_{3/2}(x)$ is a modified Bessel function. If we take $f(U)$ to be the analogue of the minimally coupled scalar, we can construct the flux factor

$$\begin{aligned} \mathcal{F} &= \lim_{U_0 \rightarrow \infty} \frac{1}{2\kappa_{10}^2} \sqrt{g} g^{UU} e^{-2(\phi - \phi_\infty)} \\ &\quad \times \partial_U \log(f(U))|_{U=U_0} \\ &= \frac{NU_0^2 k^2}{2g_{YM}^2} - \frac{N^{3/2} k^3}{4g_{YM}} + \dots \end{aligned} \quad (5)$$

up to a numerical coefficient of order one which we have suppressed. We see that the leading non-analytic contribution in k^2 is due to the k^3 term. Fourier transforming the latter yields

$$\langle \mathcal{O}(x) \mathcal{O}(0) \rangle = \frac{N^{\frac{3}{2}}}{g_{YM} x^5}. \quad (6)$$

This is in line with the discussion at the beginning of this section. We expect to deviate from the trivial $(1/x^4)$ scaling behavior of the correlator at $x_1 = 1/g_{YM}\sqrt{N_c}$ and $x_2 = \sqrt{N_c}/g_{YM}$. This yields the phase diagram in Fig. 1. It is interesting to note that the entire N_c hierarchy is consistent in the sense of Zamolodchikov's c-theorem, which assures that the central charges obey $c(x) > c(y)$, whenever $x < y$ [21].

3. The correlator from SDLCQ

Discretized Light-Cone Quantization (DLCQ) preserves supersymmetry at every stage of the

calculation if the supercharge rather than the Hamiltonian is diagonalized [4]. The framework of supersymmetric DLCQ (SDLCQ) allows one to use the advantages of light-cone quantization (*e.g.* a simpler vacuum) together with the excellent renormalization properties guaranteed by supersymmetry. Using SDLCQ, we can reproduce the SUGRA scaling relation, Eq. (6), fix the numerical coefficient, and calculate the cross-over behavior at $1/g_{YM}\sqrt{N_c} < x < \sqrt{N_c}/g_{YM}$. To exclude subtleties, *nota bene* issues of zero modes, we checked our results against the free fermion and the 't Hooft model and found consistent results.

The technique of (S)DLCQ was reviewed in Ref. [15], so we can be brief here. The basic idea of light-cone quantization is to parameterize space-time using light-cone coordinates

$$x^\pm \equiv \frac{1}{\sqrt{2}} (x^0 \pm x^1), \quad (7)$$

and to quantize the theory making x^+ play the role of time. In the discrete light-cone approach, we require the momentum $p_- = p^+$ along the x^- direction to take on discrete values in units of p^+/K where p^+ is the conserved total momentum of the system. The integer K is the so-called harmonic resolution, and plays the role of a discretization parameter. One can think of this discretization as a consequence of compactifying the x^- coordinate on a circle with a period $2L = 2\pi K/p^+$. The advantage of discretizing on the light cone is the fact that the dimension of the Hilbert space becomes finite. Therefore, the Hamiltonian is a finite-dimensional matrix, and its dynamics can be solved explicitly. In SDLCQ one makes the DLCQ approximation to the supercharges Q^i . Surprisingly, also the discrete representations of Q_i satisfy the supersymmetry algebra. Therefore SDLCQ enjoys the improved renormalization properties of supersymmetric theories. To recover the continuum result, K has to go to infinity. Incidentally, what one finds is that SDLCQ usually converges faster than the naive DLCQ towards the continuum limit.

Let us now return to the problem at hand. We would like to compute a general expression for the correlator of the form $F(x^-, x^+) =$

$\langle \mathcal{O}(x^-, x^+) \mathcal{O}(0, 0) \rangle$. In DLCQ one fixes the total momentum in the x^- direction, and it is natural to compute the Fourier transform and express it in a spectrally decomposed form

$$\begin{aligned} \tilde{F}(P_-, x^+) &= \frac{1}{2L} \langle \mathcal{O}(P_-, x^+) \mathcal{O}(-P_-, 0) \rangle \\ &= \sum_n \frac{1}{2L} \langle 0 | \mathcal{O}(P_-) | n \rangle e^{-iP_+^n x^+} \\ &\quad \times \langle n | \mathcal{O}(-P_-, 0) | 0 \rangle. \end{aligned} \quad (8)$$

The form of the correlation function in position space is then recovered by Fourier transforming with respect to $P_- = K\pi/L$. We can continue to Euclidean space by taking $r = \sqrt{2x^+x^-}$ to be real. The result for the correlator of the stress-energy tensor is

$$\begin{aligned} F(x^-, x^+) &= \sum_n \left| \frac{L}{\pi} \langle n | T^{++}(-K) | 0 \rangle \right|^2 \left(\frac{x^+}{x^-} \right)^2 \\ &\quad \times \frac{M_n^4}{8\pi^2 K^3} K_4 \left(M_n \sqrt{2x^+x^-} \right), \end{aligned} \quad (9)$$

where M_i is a mass eigenvalue and $K_4(x)$ is the modified Bessel function of order 4. Note that this quantity depends on the harmonic resolution K , but involves no other unphysical quantities. In particular, the expression is independent of the box length L .

The momentum operator $T^{++}(x)$ of two-dimensional $\mathcal{N} = 8$ SYM is given by

$$\begin{aligned} T^{++}(x) &= \text{tr} \left[(\partial_- X^I)^2 \right. \\ &\quad \left. + \frac{1}{2} (iu^\alpha \partial_- u^\alpha - i(\partial_- u^\alpha) u^\alpha) \right], \end{aligned} \quad (10)$$

with $I, \alpha = 1 \dots 8$. X and u are the physical adjoint scalars and fermions, respectively [2]. When discretized, these operators have the mode expansions

$$X_{i,j}^I = \frac{1}{\sqrt{4\pi}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \quad (11)$$

$$\begin{aligned} &\left[a_{ij}^I(n) e^{-i\pi n x^- / L} + a_{ji}^{\dagger I}(n) e^{i\pi n x^- / L} \right], \\ u_{i,j}^\alpha &= \frac{1}{\sqrt{4L}} \sum_{n=1}^{\infty} \quad (12) \\ &\left[b_{ij}^\alpha(n) e^{-i\pi n x^- / L} + b_{ji}^{\dagger \alpha}(-n) e^{i\pi n x^- / L} \right]. \end{aligned}$$

The matrix element $(L/\pi) \langle 0 | T^{++}(K) | i \rangle$ can be substituted directly to give an explicit expression for the two-point function. We see immediately that the correlator has the correct small- r behavior, for in that limit, it asymptotes to

$$\left(\frac{x^-}{x^+} \right)^2 F(x^-, x^+) = \frac{N_c^2(2n_b + n_f)}{4\pi^2 r^4} \left(1 - \frac{1}{K} \right),$$

which we expect for the theory of $n_b(n_f)$ free bosons (fermions) at large K .

On the other hand, the contribution to the correlator from strictly massless states is given by

$$\begin{aligned} \left(\frac{x^-}{x^+} \right)^2 F(x^-, x^+) &= \frac{6}{K^3 \pi^2 r^4} \quad (13) \\ &\quad \times \sum_i \left| \frac{L}{\pi} \langle 0 | T^{++}(K) | i \rangle \right|_{M_i=0}^2. \end{aligned}$$

It is important to notice that this $1/r^4$ behavior at large r is *not* the one we are looking for at large r . First of all, we do not expect any massless physical bound state in this theory, and, additionally, it has the wrong N_c dependence. Relative to the $1/r^4$ behavior at small r , the $1/r^4$ behavior at large r that we expect is down by a factor of $1/N_c$. This behavior is suppressed because we are performing a large- N_c calculation. All we can hope is to see the transition from the $1/r^4$ behavior at small r to the region where the correlator behaves like $1/r^5$.

4. Symmetries and Numerics

In principle, we can now calculate the correlator numerically by evaluating Eq. (9). However, it turns out that even for very modest harmonic resolutions, we face a tremendous numerical task. At $K = 2, 3, 4$, the dimension of the associated Fock space is 256, 1632, and 29056, respectively. The usual procedure is to diagonalize the Hamiltonian P^- and then to evaluate the projection of each eigenfunction on the fundamental state $T^{++}(-K)|0\rangle$. Since we are only interested in states which have nonzero value of such projection, we are able to significantly reduced our numerical efforts.

In the continuum limit, the result does not depend on which of the eight supercharges Q_α^- one

chooses. In DLCQ, however, the situation is a bit subtler: while the spectrum of $(Q_\alpha^-)^2$ is the same for all α , the wave functions depend on the choice of supercharge [2]. This dependence is an artifact of the discretization and disappears in the continuum limit. What happens if we just pick one supercharge, say Q_1^- ? Since the state $T^{++}(-K)|0\rangle$ is a singlet under R-symmetry acting on the “flavor” index of Q_α^- , the correlator (9) does not depend on the choice of α even at finite resolution!

We can exploit this fact to simplify our calculations. Consider an operator S commuting with both P^- and $T^{++}(-K)$, and such that $S|0\rangle = s_0|0\rangle$. Then the Hamiltonian and S can be diagonalized simultaneously. We assume in the sequel that the set of states $|i\rangle$ is a result of such a diagonalization. In this case, only states satisfying the condition $S|i\rangle = s_0|i\rangle$ contribute to the sum in (9), and we only need to diagonalize P^- in this sector, which reduces the size of the problem immensely. We can deduce from the structure of the state $T^{++}(-K)|0\rangle$ that any transformation of the form

$$\begin{aligned} a_{ij}^I(k) &\rightarrow f(I)a_{ij}^{P[I]}(k), & f(I) &= \pm 1 \\ b_{ij}^\alpha(k) &\rightarrow g(\alpha)b_{ij}^{Q[\alpha]}(k), & g(\alpha) &= \pm 1 \end{aligned} \quad (14)$$

given arbitrary permutations P and Q of the 8 flavor indices, commutes with $T^{++}(-K)$. The vacuum will then be an eigenstate of this transformation with eigenvalue 1. The requirement for $P^- = (Q_1^-)^2$ to be invariant under S imposes some restrictions on the permutations. In fact, we will require that Q_1^- be invariant under S , in order to guarantee that P^- is invariant.

The form of the supercharge from [2] is

$$\begin{aligned} Q_\alpha^- &= \int_0^\infty [\dots] b_\alpha^\dagger(k_3) a_I(k_1) a_I(k_2) + \dots \\ &+ (\beta_I \beta_J^T - \beta_J \beta_I^T)_{\alpha\beta} [\dots] b_\beta^\dagger(k_3) a_I(k_1) a_J(k_2) + \dots \end{aligned} \quad (15)$$

Here the β_I are 8×8 real matrices satisfying $\{\beta_I, \beta_J^T\} = 2\delta_{IJ}$. We use the special representation for these matrices given in Ref. [19].

Let us consider the expression for Q_1^- , Eq. (15). The first part of the supercharge does not include β matrices, and is therefore invariant under the transformation, Eq. (14), as long as $g(1) = 1$ and

$Q[1] = 1$. We will consider only such transformations. The crucial observation for the analysis of the symmetries of the β terms is that in the representation of the β matrices we have chosen, the expression $\mathcal{B}_{IJ}^\alpha = (\beta_I \beta_J^T - \beta_J \beta_I^T)_{1\alpha}$ may take only the values ± 2 or zero. Besides, for any pair (I, J) there is only one (or no) value of α corresponding to nonzero \mathcal{B} . Using this information, we may represent \mathcal{B} in a compact form. With the definition [22]

$$\mu_{IJ} = \begin{cases} \alpha, & \mathcal{B}_{IJ}^\alpha = 2 \\ -\alpha, & \mathcal{B}_{IJ}^\alpha = -2 \\ 0, & \mathcal{B}_{IJ}^\alpha = 0 \text{ for all } \alpha \end{cases}, \quad (16)$$

together with the special choice of β matrices we get the following expression for μ

$$\mu = \begin{pmatrix} 0 & 5 & -7 & 2 & -6 & 3 & -4 & 8 \\ -5 & 0 & -3 & 6 & 2 & -7 & 8 & 4 \\ 7 & 3 & 0 & -8 & -4 & -5 & 6 & 2 \\ -2 & -6 & 8 & 0 & -5 & 4 & 3 & 7 \\ 6 & -2 & 4 & 5 & 0 & -8 & -7 & 3 \\ -3 & 7 & 5 & -4 & 8 & 0 & -2 & 6 \\ 4 & -8 & -6 & -3 & 7 & 2 & 0 & 5 \\ -8 & -4 & -2 & -7 & -3 & -6 & -5 & 0 \end{pmatrix}.$$

The next step is to look for a subset of the transformations, Eq. (14), which satisfy the conditions $g(1) = 1$ and $Q[1] = 1$ and leave the matrix μ invariant. This invariance implies that

$$Q[\mu_{P[I]P[J]}] = g(\mu_{IJ})f(I)f(J)\mu_{IJ}. \quad (17)$$

The subset of transformations we are looking for forms a subgroup R of the permutation group $S_8 \times S_8$. Consequently, we will search for the elements of R that square to one. Products of such elements generate the whole group in the case of $S_8 \times S_8$. We will show later that this remains true for R . Not all of the Z_2 symmetries satisfying (17) are independent. In particular, if a and b are two such symmetries then aba is also a valid Z_2 symmetry. By going systematically through the different possibilities, we have found that there are 7 independent Z_2 symmetries in the group R . They are listed in Table 1. We explicitly constructed all the symmetries of the type, Eq. (14), which satisfy Eq. (17) using MATHEMATICA. It turns out that the group of such transformations has 168 elements, and we have shown that all of them can be generated from the seven Z_2 symmetries mentioned above.

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	b_2	b_3	b_4	b_5	b_6	b_7	b_8
1	a_7	a_3	a_2	a_6	a_8	a_4	a_1	a_5	b_2	$-b_3$	$-b_4$	$-b_6$	$-b_5$	b_8	b_7
2	a_3	a_6	a_1	a_5	a_4	a_2	a_8	a_7	$-b_4$	b_3	$-b_2$	$-b_5$	b_8	$-b_7$	b_6
3	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	$-b_3$	$-b_2$	b_4	$-b_5$	b_7	b_6	$-b_8$
4	a_5	a_4	a_8	a_2	a_1	a_7	a_6	a_3	$-b_2$	$-b_7$	b_8	b_5	$-b_6$	$-b_3$	b_4
5	a_8	a_3	a_2	a_7	a_6	a_5	a_4	a_1	$-b_5$	$-b_3$	b_7	$-b_2$	b_6	b_4	$-b_8$
6	a_5	a_8	a_7	a_6	a_1	a_4	a_3	a_2	$-b_8$	b_5	$-b_4$	b_3	$-b_6$	b_7	$-b_2$
7	a_4	a_6	a_8	a_1	a_7	a_2	a_5	a_3	$-b_2$	$-b_6$	b_5	b_4	$-b_3$	$-b_7$	b_8

Table 1

Seven independent Z_2 symmetries of the group R , which act on the 'flavor' quantum number of the different particles. Under the first of these symmetries, *e.g.*, the boson a_1 is transformed into a_7 , etc.

In our numerical algorithm we implemented the Z_2 symmetries as follows. We can group the Fock states in classes and treat the whole class as a new state, because all states relevant for the correlator are singlets under the symmetry group R . as an example consider the simplest non-trivial singlet

$$|1\rangle = \frac{1}{8} \sum_{I=1}^8 \text{tr} (a^\dagger(1, I) a^\dagger(K-1, I)) |0\rangle. \quad (18)$$

Hence, if we encounter the state $a^\dagger(1, 1) a^\dagger(K-1, 1) |0\rangle$ while constructing the basis, we will replace it by the class representative; in this case, by the state $|1\rangle$. Such a procedure significantly decreases the size of the basis, while keeping all the information necessary for calculating the correlator. In summary, this use of the discrete flavor symmetry of the problem reduces the size of the Fock space by orders of magnitude.

In addition to these simplifications, one can further improve on the numerical efficiency by using Lanczos diagonalization techniques. Namely, we substitute the explicit diagonalization with an efficient approximation. The idea is to use a symmetry preserving (Lanczos) algorithm. If we start with a normalized vector $|u_1\rangle$ proportional to the fundamental state $T^{++}(-K)|0\rangle$, the Lanczos recursion will produce a tridiagonal representation of the Hamiltonian $H_{LC} = 2P^+P^-$. Due to orthogonality of $\{|u_i\rangle\}$, only the (1,1) element of the tridiagonal matrix, $\hat{H}_{1,1}$, will contribute to the correlator. We exponentiate by diagonalizing $\hat{H}_{LC} \vec{v}_i = \lambda_i \vec{v}_i$ with eigenvalues λ_i and get

$$F(P^+, x^+) = \frac{|N_0|^{-2}}{2L} \left(\frac{\pi}{L}\right)^2 \sum_{j=1}^{N_L} |(v_j)_1|^2 e^{-i \frac{\lambda_j L}{2K\pi} x^+}.$$

Finally, we Fourier transform to obtain

$$F(x^-, x^+) = \frac{1}{8\pi^2 K^3} \left(\frac{x^+}{x^-}\right)^2 \frac{1}{|N_0|^2} \quad (19)$$

$$\times \sum_{j=1}^{N_L} |(v_j)_1|^2 \lambda_j^2 K_4(\sqrt{2x^+ x^- \lambda_i}),$$

which is equivalent to Eq. (9). This algorithm is correct only if the number of Lanczos iterations N_L runs up to the rank of original matrix. But *in praxi* already a basis of about 20 vectors covers all leading contribution to correlator [13].

5. Results

To evaluate the expression for the correlator $\mathcal{F}(r)$, we have to calculate the mass spectrum and insert it into Eq. (9). In the $\mathcal{N} = (8, 8)$ supersymmetric Yang-Mills theory the contribution of massless states becomes a problem. These states exist in the SDLCQ calculation, but are unphysical. It has been shown that these states are not normalizable and that the number of partons in these states is even (odd) for K even (odd) [2]. Because the correlator is only sensitive to two particle contributions, the curves $\mathcal{F}(r)$ are different for even and odd K . Unfortunately, the unphysical states yield also the typical $1/r^4$ behavior, but have a wrong N_c dependence. The regular $1/r^4$ contribution is down by $1/N_c$, so we cannot see this contribution at large r , because we are working in the large N_c limit.

We can use this information about the unphysical states, however, to determine when our approximation breaks down. It is the region where

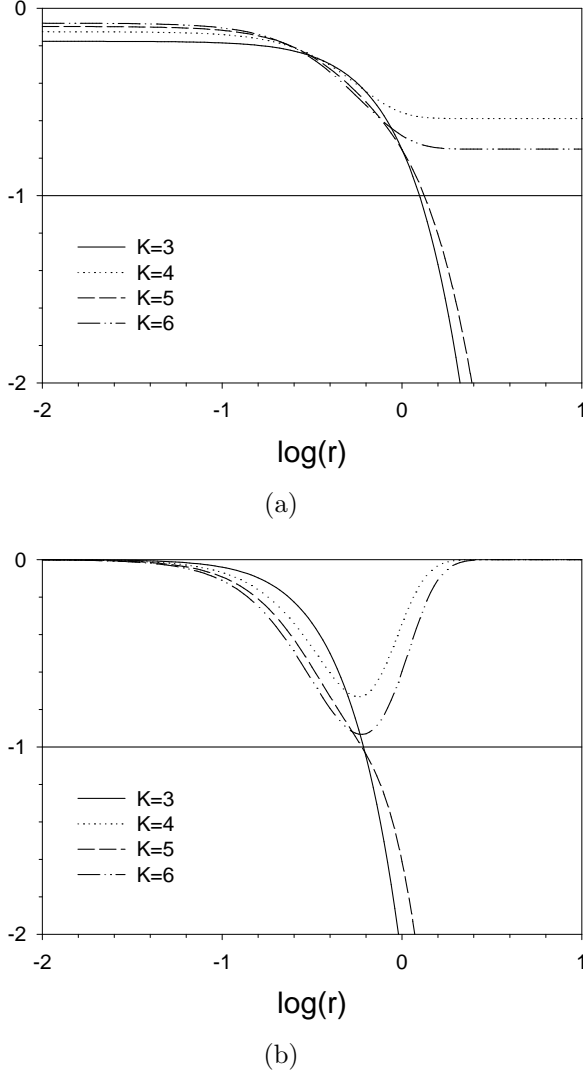


Figure 2. Top: (a) Log-Log plot of $\mathcal{F}(r) = \langle T^{++}(x)T^{++}(0) \rangle \left(\frac{x^-}{x^+} \right)^2 \frac{4\pi^2 r^4}{N_c^2(2n_b+n_f)}$ vs. r for $g_{YM}^2 N_c/\pi = 1.0$, $K = 3, 4, 5$ and 6 . Bottom: (b) the log-log derivative with respect to r of the correlation function in (a).

the unphysical massless states dominate the correlator sum. Unfortunately, this is also the region where we expect the true large- r behavior to dominate the correlator, if only the extra states were absent. In Fig. 2(a) for even resolution, the region where the correlator starts to behave like $1/r^4$ at large r is clearly visible. In Fig. 2(b) we see that for even resolution the effect of the massless state on the derivative is felt at smaller values of r where the even resolution curves start to turn up. Another estimate of where this approximation breaks down, that gives consistent values, is the set of points where the even and odd resolution derivative curves cross. We do not expect these curves to cross on general grounds, based on work in [3], where we considered a number of other theories. Our calculation is consistent in the sense that this breakdown occurs at larger and larger r as K grows.

We expect to approach the line $d\mathcal{F}(r)/dr = -1$ line signaling the cross-over from the trivial $1/r^4$ behavior to the characteristic $1/r^5$ behavior of the supergravity correlator, Eq. (6). Indeed, the derivative curves in Fig. 2(b) are approaching -1 as we increase the resolution and appear to be about 85 – 90% of this value before the approximation breaks down. There is, however, no indication of convergence yet; therefore, we cannot claim a numerical proof of the Maldacena conjecture. A safe signature of equivalence of the field and string theories would be if the derivative curve would flatten out at -1 before the approximation breaks down.

6. Conclusions

In this note we reported on progress in an attempt to rigorously test the conjectured equivalence of two-dimensional $\mathcal{N} = (8, 8)$ supersymmetric Yang-Mills theory and a system of $D1$ -branes in string theory. Within a well-defined non-perturbative calculation, we obtained results that are within 10-15% of results expected from the Maldacena conjecture. The results are still not conclusive, but they definitely point in right direction. Compared to previous work [3], we included orders of magnitude more states in our calculation and thus greatly improved the test-

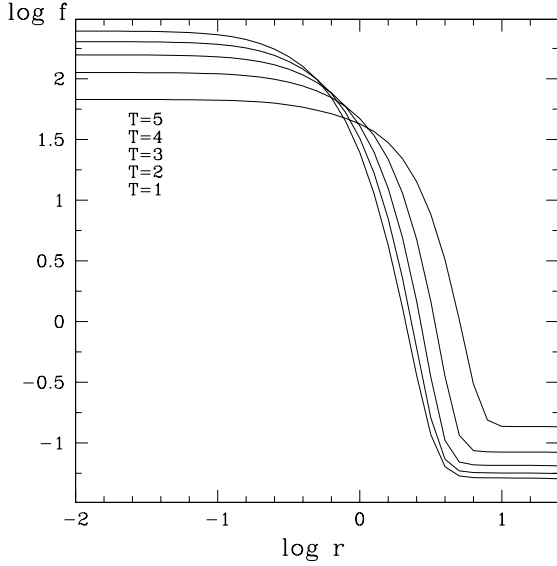


Figure 3. Log-Log plot of the *three-dimensional* correlation function $f \equiv r^5 \langle T^{++}(x) T^{++}(0) \rangle \left(\frac{x^-}{x^+} \right)^2 \frac{16\pi^3 K^3 l}{105\sqrt{-i}}$ vs. r for $g = g_{YM}^2 \sqrt{N_c} l / 2\pi^{3/2} = 1.0$ for $K = 6$ and $T = 1$ to 5.

ing conditions. We remark that improvements of the code and the numerical method are possible and under way. During the calculation we noticed that contributions to the correlator come from only a small number of terms. An analytic understanding of this phenomenon would greatly accelerate calculations. We point out that in principle we could study the proper $1/r$ behavior at large r by computing $1/N_c$ corrections, but this interesting calculation would mean a huge numerical effort.

7. Outlook

It remains a challenge to rigorously test the conjectured string/field theory correspondences. Although the so-called Maldacena conjecture maybe the most exciting one, because it promises insight into full four-dimensional Yang-Mills theories in the strong coupling regime, there are other interesting scenarios. For instance, it was conjectured that the supergravity solutions correspond-

ing to $p + 1$ SYM theories are black p -brane solutions, see *e.g.* Ref. [5]. Consequently, there are interesting testing scenarios also in three-dimensional spacetime. Numerically, of course, things get the more difficult, the more dimensions are involved. On the way to the full four-dimensional problem, it may be worthwhile to present our latest results on correlation functions in three dimensions, see also [23]. Fig. 6 shows the correlator for $\mathcal{N} = 1$ SYM(2+1) as a function of the distance r : it is converging well with the *transverse* cut-off T . To put things in perspective, we note that the largest Hamiltonian matrix involved in this calculations requires to set up a Fock basis of approximately two million states. This is by a factor 100 more than we used in the test of the Maldacena conjecture described in this article, which itself was already substantially better than the first feasibility study [3].

We hope to proceed on this way and to be able to present interesting results soon.

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